

# 9.3 Find Special Products of Polynomials



**Before**

You multiplied polynomials.

**Now**

You will use special product patterns to multiply polynomials.

**Why?**

So you can make a scientific prediction, as in Example 4.

## Key Vocabulary

- **binomial**, p. 555
- **trinomial**, p. 555

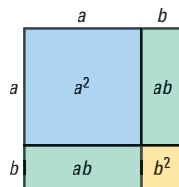
The diagram shows a square with a side length of  $(a + b)$  units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$

This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace  $b$  with  $-b$ .

$$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2 \quad \text{Replace } b \text{ with } -b \text{ in the pattern above.}$$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Simplify.}$$



## KEY CONCEPT

*For Your Notebook*

### Square of a Binomial Pattern

**Algebra**

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

**Example**

$$(x + 5)^2 = x^2 + 10x + 25$$

$$(2x - 3)^2 = 4x^2 - 12x + 9$$

## EXAMPLE 1 Use the square of a binomial pattern

### USE PATTERNS

When you use special product patterns, remember that  $a$  and  $b$  can be numbers, variables, or variable expressions.

**Find the product.**

$$\begin{aligned} \text{a. } (3x + 4)^2 &= (3x)^2 + 2(3x)(4) + 4^2 \\ &= 9x^2 + 24x + 16 \end{aligned}$$

**Square of a binomial pattern**

**Simplify.**

$$\begin{aligned} \text{b. } (5x - 2y)^2 &= (5x)^2 - 2(5x)(2y) + (2y)^2 \\ &= 25x^2 - 20xy + 4y^2 \end{aligned}$$

**Square of a binomial pattern**

**Simplify.**



### GUIDED PRACTICE for Example 1

**Find the product.**

$$1. (x + 3)^2 = x^2 + 6x + 9$$

$$2. (2x + 1)^2 = 4x^2 + 4x + 1$$

$$3. (4x - y)^2 = 16x^2 - 8xy + y^2$$

$$4. (3m + n)^2 = 9m^2 + 6mn + n^2$$

## 1 PLAN AND PREPARE

### Warm-Up Exercises

**Transparency Available**

Find the product.

$$1. (x + 7)(x + 2) \quad x^2 + 9x + 14$$

$$2. (3x - 1)(3x + 2) \quad 9x^2 + 3x - 2$$

3. The dimensions of a rectangular playground can be represented by  $3x + 8$  and  $5x + 2$ . Write a polynomial that represents the area of the playground. What is the area of the playground if  $x$  is 8 meters?  $15x^2 + 46x + 16$ ;  $1344 \text{ m}^2$

### Notetaking Guide

**Transparency Available**

Promotes interactive learning and notetaking skills, pp. 195–198.

### Pacing

**Basic:** 1 day

**Average:** 1 day

**Advanced:** 1 day

**Block:** 0.5 block with 9.4

• See *Teaching Guide/Lesson Plan*.

## 2 FOCUS AND MOTIVATE

### Essential Question

**Big Idea 1**, p. 553

How do you use special product patterns to multiply binomials? **Tell students they will learn how to answer this question by using patterns to write products of binomials.**

### NCTM STANDARDS

**Standard 2:** Understand patterns; Use models to understand relationships

## Resource Planning Guide

### Chapter Resource Book

- Teaching Guide/Lesson Plan (pp. 26–27)
- Practice levels A, B, C (pp. 28–30)
- Study Guide (pp. 31–32)
- Catch-up for Absent Students (p. 33)
- Application (p. 34)
- Challenge (p. 35)

### Workbooks

- Notetaking Guide (pp. 195–198)
- Practice Workbook (pp. 137–138)

### Teaching Options

- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

### Interactive Technology

- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

### Resources for English Learners

- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the *Algebra 1 Toolkit* for more strategies for meeting individual needs.

## Motivating the Lesson

If both parents have brown eyes, it is possible for one child to have brown eyes while another child has blue eyes. By using a special pattern to multiply binomials, you can calculate the likelihood of such an occurrence.

## 3 TEACH

### Extra Example 1

Find the product.

a.  $(2x + 5)^2$   $4x^2 + 20x + 25$

b.  $(3x - y)^2$   $9x^2 - 6xy + y^2$

### Key Question to Ask for Example 1

- How would the product  $(3x - 4)^2$  be different from the product in part (a)? **The middle term would be negative rather than positive.**

### Extra Example 2

Find the product.

a.  $(r + 3)(r - 3)$   $r^2 - 9$

b.  $(4x + y)(4x - y)$   $16x^2 - y^2$

### Key Question to Ask for Example 2

- What happens to the middle terms when you find the product using the sum and difference pattern? **The sum of the two middle terms is zero.**

### Extra Example 3

Use special products to find the product  $18 \cdot 22$ . **396**

**SUM AND DIFFERENCE PATTERN** To find the product  $(x + 2)(x - 2)$ , you can multiply the two binomials using the FOIL pattern.

$$\begin{aligned}(x + 2)(x - 2) &= x^2 - 2x + 2x - 4 && \text{Use FOIL pattern.} \\ &= x^2 - 4 && \text{Combine like terms.}\end{aligned}$$

This suggests a pattern for the product of the sum and difference of two terms.

## KEY CONCEPT

*For Your Notebook*

### Sum and Difference Pattern

Algebra

$$(a + b)(a - b) = a^2 - b^2$$

Example

$$(x + 3)(x - 3) = x^2 - 9$$

## EXAMPLE 2 Use the sum and difference pattern

Find the product.

a.  $(t + 5)(t - 5) = t^2 - 5^2$  **Sum and difference pattern**  
 $= t^2 - 25$  **Simplify.**

b.  $(3x + y)(3x - y) = (3x)^2 - y^2$  **Sum and difference pattern**  
 $= 9x^2 - y^2$  **Simplify.**



## GUIDED PRACTICE for Example 2

Find the product.

5.  $(x + 10)(x - 10)$   
 $x^2 - 100$

6.  $(2x + 1)(2x - 1)$   
 $4x^2 - 1$

7.  $(x + 3y)(x - 3y)$   
 $x^2 - 9y^2$

**SPECIAL PRODUCTS AND MENTAL MATH** The special product patterns can help you use mental math to find certain products of numbers.

## EXAMPLE 3 Use special products and mental math

Use special products to find the product  $26 \cdot 34$ .

**Solution**

Notice that 26 is 4 less than 30 while 34 is 4 more than 30.

$$\begin{aligned}26 \cdot 34 &= (30 - 4)(30 + 4) && \text{Write as product of difference and sum.} \\ &= 30^2 - 4^2 && \text{Sum and difference pattern} \\ &= 900 - 16 && \text{Evaluate powers.} \\ &= 884 && \text{Simplify.}\end{aligned}$$

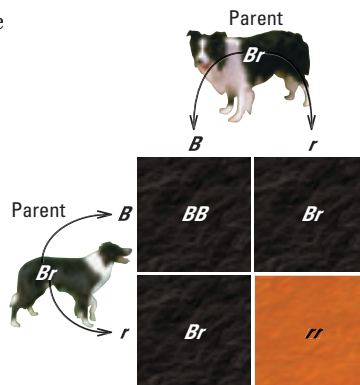
## Differentiated Instruction

**Visual Learners** Students sometimes forget the coefficient of the linear term  $ab$  in the square of a binomial, or they misplace the minus sign. In addition to the colored tiles shown on page 569, another way to remember the formulas is to apply the FOIL method to the product of two binomials using mental math. See also the *Algebra 1 Toolkit* for more strategies.

### EXAMPLE 4 Solve a multi-step problem

**BORDER COLLIES** The color of the dark patches of a border collie's coat is determined by a combination of two genes. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene  $B$  is for black patches, and the gene  $r$  is for red patches. Any gene combination with a  $B$  results in black patches. Suppose each parent has the same gene combination  $Br$ . The Punnett square shows the possible gene combinations of the offspring and the resulting patch color.



- What percent of the possible gene combinations of the offspring result in black patches?
- Show how you could use a polynomial to model the possible gene combinations of the offspring.

#### Solution

**STEP 1 Notice** that the Punnett square shows 4 possible gene combinations of the offspring. Of these combinations, 3 result in black patches.

▶ 75% of the possible gene combinations result in black patches.

**STEP 2 Model** the gene from each parent with  $0.5B + 0.5r$ . There is an equal chance that the collie inherits a black or red gene from each parent.

The possible genes of the offspring can be modeled by  $(0.5B + 0.5r)^2$ . Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

$$\begin{aligned}(0.5B + 0.5r)^2 &= (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2 \\ &= 0.25B^2 + 0.5Br + 0.25r^2\end{aligned}$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

25%  $BB$ ,  
black patches    
 50%  $Br$ ,  
black patches    
 25%  $rr$ ,  
red patches

The coefficients show that  $25\% + 50\% = 75\%$  of the possible gene combinations will result in black patches.

#### GUIDED PRACTICE for Examples 3 and 4

- Describe how you can use special products to find  $21^2$ . Use the square of a binomial pattern to find the product  $(20 + 1)^2$ .
- BORDER COLLIES** Look back at Example 4. What percent of the possible gene combinations of the offspring result in red patches? **25%**

### Extra Example 4

In dogs, the gene  $E$  is for erect ears and the gene  $e$  is for droopy ears. Any gene combination with an  $E$  results in erect ears. The Punnett square shows the possible gene combinations of the offspring and the resulting type of ear.

	$E$	$e$
$E$	$EE$ erect	$Ee$ erect
$e$	$Ee$ erect	$ee$ droopy

- What percent of the possible gene combinations of the offspring result in droopy ears? **25%**
- Show how you could use a polynomial to model the possible gene combinations of the offspring. Use  $(0.5E + 0.5e)^2$  to model the possible genes of the offspring. Expand the product to find the possible gene combinations:  $0.25E^2 + 0.5Ee + 0.25e^2$ . The coefficient of  $e^2$  shows that 25% of the possible gene combinations will result in droopy ears.

### Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you use special product patterns to multiply binomials?

- The square of a binomial pattern is for  $(a + b)^2$  or  $(a - b)^2$ .
- The sum and difference pattern is for  $(a + b)(a - b)$ .

To find  $(a + b)^2$  or  $(a - b)^2$ , square  $a$ , add (or subtract) twice the product  $ab$ , and add the square of  $b$ . The product  $(a + b)(a - b)$  is  $a^2 - b^2$ .