Avoiding Common Errors

Exercises 7–18 Some students' graphs may cross the x-axis. Remind these students that graphs of exponential functions get closer and closer to the x-axis, but do not cross it.

Graphing Calculator

Exercises 20–31 Students may want to use their graphing calculators to compare the graphs. Remind them to use parentheses around the fractions and carets to indicate the exponent.

Animated Algebra classzone.com

An **Animated Algebra** activity is available on-line for **Exercise 38**. This activity is also available on the **Power Presentations CD-ROM**.

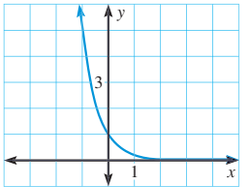
35. initial amount: 90,000 people, decay factor: 0.975, decay rate: 2.5%; Let P represent the population and t represent the number of years. $P = 90{,}000(0.975)^t$

45. Sample answer: Since the quantity loses the same percent each time period, the amount y remaining after x time periods can be modeled using the exponential function $y = a(1 - r)^x$. After 1 time period, when x is 1, $y = a(1 - r)^1$, or $a(1 - r)$.

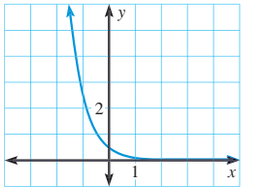
B **MATCHING** **Match the function with its graph.**

32. $y = (0.2)^x$ **A** **33.** $y = 5(0.2)^x$ **C** **34.** $y = \frac{1}{2}(0.2)^x$ **B**

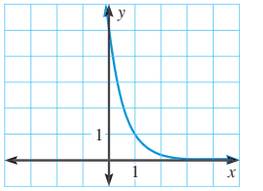
A.



B.



C.



35. POPULATION A population of 90,000 decreases by 2.5% per year. Identify the initial amount, the decay factor, and the decay rate. Then write a function that models the population over time. **See margin.**

36. ★ MULTIPLE CHOICE What is the decay rate of the function $y = 4(0.97)^t$? **D**

Ⓐ 4 Ⓑ 0.97 Ⓒ 0.3 Ⓓ 0.03

37. The decay rate, r, is 0.14. So the decay factor (1 − r) should be 0.86, not 0.14; $y = 25{,}000(0.86)^t$.

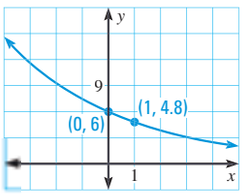
37. ERROR ANALYSIS In 2004 a person purchased a car for $25,000. The value of the car decreased by 14% annually. *Describe* and correct the error in writing a function that models the value of the car since 2004.

$y = a(1 - r)^t$
$= 25{,}000(0.14)^t$ ✗

EXAMPLE 4 on p. 533 for Exs. 38–40

RECOGNIZING EXPONENTIAL MODELS **Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.**

38.



38. exponential decay; $y = 6 \cdot 0.8^x$

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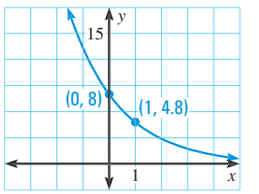
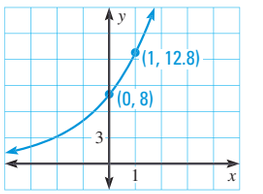
39.



exponential decay; $y = 8 \cdot 0.6^x$

40.



exponential growth; $y = 8 \cdot 1.6^x$

41b. The graph is a vertical stretch with a reflection in the x-axis.

41. REASONING Without graphing, explain how the graphs of the given functions are related to the graph of $f(x) = (0.5)^x$.

a. $m(x) = \frac{1}{3} \cdot (0.5)^x$ **The graph is a vertical shrink.**

b. $n(x) = -4 \cdot (0.5)^x$

c. $p(x) = (0.5)^x + 1$

41c. The graph is a vertical shift up 1 unit.

C **CHALLENGE** **Write an exponential function of the form $y = ab^x$ whose graph passes through the given points.**

42. $(0, 1), \left(2, \frac{1}{4}\right)$ $y = \left(\frac{1}{2}\right)^x$ **43.** $(1, 20), (2, 4)$ $y = 100 \cdot \left(\frac{1}{5}\right)^x$ **44.** $\left(1, \frac{3}{2}\right), \left(2, \frac{3}{4}\right)$ $y = 3 \cdot \left(\frac{1}{2}\right)^x$

45. ★ WRITING The initial amount of a quantity is a units and the quantity is decaying at a rate of r (a percent per time period). Show that the amount of the quantity after one time period is $a(1 - r)$. *Explain* how you found your answer. **See margin.**

46. Sample answer: The graphs are the same. By the product of powers property, $4^{x-2} = 4x \cdot 4^{-2}$, or $\frac{1}{16} \cdot 4x$.

46. CHALLENGE Compare the graph of the function $f(x) = 4^{x-2}$ with the graph of the function $g(x) = \frac{1}{16} \cdot 4^x$. Use properties of exponents to explain your observation.

○ = WORKED-OUT SOLUTIONS for on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

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PROBLEM SOLVING

GRAPHING CALCULATOR **You may wish to use a graphing calculator to complete the following Problem Solving exercises.**

EXAMPLE 5 A on p. 534 for Exs. 47–50

47. CELL PHONES You purchase a cell phone for $125. The value of the cell phone decreases by about 20% annually. Write a function that models the value of the cell phone over time. Then find the value of the cell phone after 3 years. Round to the nearest dollar.

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Let V represent the value of the cell phone and t represent the number of years since purchase, $V = 125(0.8)^t$; $64.

48. ANIMAL POPULATION Scientists studied the population of a species of bat in some caves in Missouri from 1983 to 2003. In 1983, there were 141,200 bats living in the caves. That number decreased by about 11% annually until 2003.

a. Identify the initial amount, the decay factor, and the decay rate.

b. Write a function that models the number of bats since 1983. Then find the number of bats in 2003.

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48a. initial amount: 141,200; decay factor: 0.89; decay rate: 11%

48b. Let B represent the number of bats and t represent the number of years since 1983, $B = 141{,}200\,(0.89)^t$; 13,729 bats.

49. ★ **SHORT RESPONSE** In 2003 a family bought a boat for $4000. The boat depreciates (loses value) at a rate of 7% annually. In 2006 a person offers to buy the boat for $3000. Should the family sell the boat? *Explain*.

No. Sample answer: The boat's value is worth about $3217.

50. **MULTIPLE REPRESENTATIONS** There are a total of 128 teams at the start of a citywide 3-on-3 basketball tournament. Half of the teams are eliminated after each round.

a. Writing a Model Write a function for the number of teams left after x rounds. $f(x) = 128(0.5)^x$

b. Making a Table Make a table for the function using $x = 0, 1, 2, \ldots, 7$. See margin.

c. Drawing a Graph Use the table in part (b) to graph the function. After which round are there 4 teams left in the tournament?

See margin for art; round 5.

B

51. GUITARS The frets on a guitar are the small metal bars that divide the fingerboard. The distance d (in inches) between the nut and the first fret or any two consecutive frets can be modeled by the function $d = 1.516(0.9439)^f$ where f is the number of the fret farthest from the nut.

51a. decay factor: 0.9439, decay rate: 5.61%

a. Identify the decay factor and the decay rate for the model.

b. What is the distance between the nut and the first fret? about 1.431 in.

c. The distance between the 12th and 13th frets is about half the distance between the nut and the first fret. Use this fact to find the distance between the 12th and 13th frets. Use the model to verify your answer. about 0.716 in.

Study Strategy

Exercise 48 Suggest that students use the exponential decay model on page 534 to check their solutions to parts (a) and (b).

Vocabulary

Exercise 49 You may want to ask a student to use a dictionary or thesaurus to define the word depreciate. Have the student look up the word appreciate as an antonym to depreciate. Then ask them to discuss the relationship of these terms to exponential functions.

Internet Reference

Exercise 53 More information about maximal oxygen consumption can be found at www.nismat.org/physcor/max_o2.html

50b.

Rounds completed	Teams remaining
0	128
1	64
2	32
3	16
4	8
5	4
6	2
7	1

50c.