Extra Example 5

A farmer bought a tractor in 1999 for $30,000. The value of the tractor has been decreasing at a rate of 18% per year.

a. Write a function that models the value of the tractor over time. **$V = 30{,}000(1 - 0.18)^t$**

b. What was the approximate value of the tractor in 2005? **about $9120**

Key Question to Ask for Example 5

- Why do you subtract the decay rate from 1 in the exponential decay model? **Because the acreage is decreasing, the decay factor is less than 1.**

Animated Algebra classzone.com

An **Animated Algebra** activity is available on-line for **Example 5**. This activity is also available on the **Power Presentations CD-ROM**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you write and graph exponential decay functions?

- **The function $y = ab^x$ represents exponential decay when $a > 0$ and $0 < b < 1$.**
- **The exponential decay model is $y = a(1 - r)^t$, where a is the initial amount, $1 - r$ is the decay factor, and t is the time period.**

To write a function rule, use the form $y = ab^x$. Substitute the y-intercept for a and then use another point from the table to solve the equation for b. To graph a function of the form $y = ab^x$, create a table of values, plot the points, and connect the points with a smooth curve.

EXPONENTIAL DECAY When a quantity decays exponentially, it decreases by the same percent over equal time periods. To find the amount of the quantity left after t time periods, use the following model.

KEY CONCEPT *For Your Notebook*

Exponential Decay Model

a is the **initial amount**. r is the **decay rate**.

$$y = a(1 - r)^t$$

$1 - r$ is the **decay factor**. t is the **time period**.

REWRITE EQUATIONS

Notice that you can rewrite $y = ab^x$ as $y = a(1 - r)^t$ by replacing b with $1 - r$ and x with t (for time).

The relationship between the decay rate r and the decay factor $1 - r$ is similar to the relationship between the growth rate and growth factor in an exponential growth model. You will explore this relationship in Exercise 45.

EXAMPLE 5 Solve a multi-step problem

FORESTRY The number of acres of Ponderosa pine forests decreased in the western United States from 1963 to 2002 by 0.5% annually. In 1963 there were about 41 million acres of Ponderosa pine forests.

a. Write a function that models the number of acres of Ponderosa pine forests in the western United States over time.

b. To the nearest tenth, about how many million acres of Ponderosa pine forests were there in 2002?

Solution

a. Let P be the number of acres (in millions), and let t be the time (in years) since 1963. The initial value is 41, and the decay rate is 0.005.

$P = a(1 - r)^t$ Write exponential decay model.

$= 41(1 - 0.005)^t$ Substitute 41 for a and 0.005 for r.

$= 41(0.995)^t$ Simplify.

AVOID ERRORS

The decay rate in this example is 0.5%, or 0.005. So, the decay factor is $1 - 0.005$, or 0.995, not 0.005.

b. To find the number of acres in 2002, 39 years after 1963, substitute 39 for t.

$P = 41(0.995)^{39} \approx 33.7$ Substitute 39 for t. Use a calculator.

▶ There were about 33.7 million acres of Ponderosa pine forests in 2002.

Animated Algebra at classzone.com

✓ GUIDED PRACTICE for Example 5

5. WHAT IF? In Example 5, suppose the decay rate of the forests remains the same beyond 2002. About how many acres will be left in 2010? **about 32.4 million**

8.6 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7 and 49
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 36, 45, and 49
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 50

SKILL PRACTICE

A

1. **VOCABULARY** What is the decay factor in the exponential decay model $y = a(1 - r)^t$? **1 – r**

2. ★ **WRITING** *Explain* how you can tell if a graph represents *exponential growth* or *exponential decay*. **See margin.**

EXAMPLE 1 on p. 531 for Exs. 3–6

WRITING FUNCTIONS **Tell whether the table represents an exponential function. If so, write a rule for the function.**

3.

x	−1	0	1	2
y	2	8	32	128

exponential function; $y = 8 \cdot 4^x$

4.

x	−1	0	1	2
y	50	10	2	0.4

exponential function; $y = 10(0.2)^x$

5.

x	−1	0	1	2
y	6	2	$\frac{2}{3}$	$\frac{2}{9}$

exponential function; $y = 2\left(\frac{1}{3}\right)^x$

6.

x	−1	0	1	2
y	−11	−7	−3	1

not an exponential function

EXAMPLE 2 on p. 532 for Exs. 7–18

GRAPHING FUNCTIONS **Graph the function and identify its domain and range.** **7–18. See margin.**

7. $y = \left(\frac{1}{5}\right)^x$ 8. $y = \left(\frac{1}{6}\right)^x$ 9. $y = \left(\frac{2}{3}\right)^x$ 10. $y = \left(\frac{3}{4}\right)^x$

11. $y = \left(\frac{4}{5}\right)^x$ 12. $y = \left(\frac{3}{5}\right)^x$ 13. $y = (0.3)^x$ 14. $y = (0.5)^x$

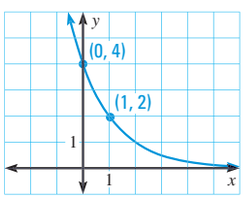
15. $y = (0.1)^x$ 16. $y = (0.9)^x$ 17. $y = (0.7)^x$ 18. $y = (0.25)^x$

EXAMPLE 3 on p. 532 for Exs. 19–31

19. ★ **MULTIPLE CHOICE** The graph of which function is shown? **D**

Ⓐ $y = (0.25)^x$ Ⓑ $y = (0.5)^x$

Ⓒ $y = 0.25 \cdot (0.5)^x$ Ⓓ $y = 4 \cdot (0.5)^x$



COMPARING FUNCTIONS **Graph the function. Compare the graph with the graph of $y = \left(\frac{1}{4}\right)^x$.** **20–31. See margin.**

20. $y = 5 \cdot \left(\frac{1}{4}\right)^x$ 21. $y = 3 \cdot \left(\frac{1}{4}\right)^x$ 22. $y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$ 23. $y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$

24. $y = 0.2 \cdot \left(\frac{1}{4}\right)^x$ 25. $y = 1.5 \cdot \left(\frac{1}{4}\right)^x$ 26. $y = -5 \cdot \left(\frac{1}{4}\right)^x$ 27. $y = -3 \cdot \left(\frac{1}{4}\right)^x$

28. $y = -\frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$ 29. $y = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$ 30. $y = -0.2 \cdot \left(\frac{1}{4}\right)^x$ 31. $y = -1.5 \cdot \left(\frac{1}{4}\right)^x$

2. Sample answer: **If the graph increases from left to right, then it represents growth; if it decreases from left to right, then it represents decay.**

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: pp. 535–538
Exs. 1–18, 63–66
Day 2: pp. 535–538
Exs. 19–35 odd, 36–40, 47–50, 54–62 even

Average:
Day 1: pp. 535–538
Exs. 1–6, 10–18, 32–34, 63–66
Day 2: pp. 535–538
Exs. 19, 20–30 even, 35–44, 47–52, 55, 58, 61

Advanced:
Day 1: pp. 535–538
Exs. 1, 4–6, 12–18, 32–37, 63–66
Day 2: pp. 535–538
Exs. 19, 24–31, 38–53*, 56, 59, 62

Block:
pp. 535–538
Exs. 1–6, 10–19, 20–30 even, 32–44, 47–52, 55, 58, 61, 63–66

Differentiated Instruction

See *Algebra 1 Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 21, 38, 47

Average: 5, 12, 24, 39, 48

Advanced: 6, 16, 28, 40, 49

Extra Practice

- Student Edition, p. 945
- Chapter 8 Resource Book: Practice levels A, B, C, pp. 66–71

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 486C.

7–18, 20–31. See Additional Answers beginning on p. AA1.