

Extra Example 4

The owner of an original copy of a 1938 comic book sold it at an auction in 2005. The owner bought the comic book for \$55 in 1980. The value of the comic book increased at a rate of 2.8% per year.

- a. Write a function that models the value of the comic book over time. $C = 55(1.028)^t$
- b. What was the approximate value of the comic book at the time of the auction in 2005? Round your answer to the nearest dollar.
about \$110

Key Questions to Ask for Example 4

- How is an exponential growth model different from a linear model? **The increase in an exponential model is the same percent every year, while the increase in a linear model is the same amount every year.**
- What types of quantities can be represented by an exponential growth model? **quantities that increase by the same percent in equal periods of time**



An **Animated Algebra** activity is available on-line for **Example 4**. This activity is also available on the **Power Presentations CD-ROM**.

REWRITE EQUATIONS

Notice that you can rewrite $y = ab^x$ as $y = a(1 + r)^t$ by replacing b with $1 + r$ and x with t (for time).

ANOTHER WAY

For alternative methods for solving Example 4, turn to page 528 for the **Problem Solving Workshop**.

AVOID ERRORS

The growth rate in this example is 6.9%, or 0.069. So, the growth factor is $1 + 0.069$, or 1.069, not 0.069.

EXPONENTIAL GROWTH When $a > 0$ and $b > 1$, the function $y = ab^x$ represents **exponential growth**. When a quantity grows exponentially, it increases by the same percent over equal time periods. To find the amount to which the quantity grows after t time periods, use the following model.

KEY CONCEPT

For Your Notebook

Exponential Growth Model

a is the **initial amount**. r is the **growth rate**.
 $y = a(1 + r)^t$
 $1 + r$ is the **growth factor**. t is the **time period**.

Notice the relationship between the growth rate r and the growth factor $1 + r$. If the initial amount of a quantity is a units and the quantity is growing at a rate of r , then after one time period the new amount is:

$$\text{Initial amount} + \text{amount of increase} = a + r \cdot a = a(1 + r)$$

EXAMPLE 4 Solve a multi-step problem

COLLECTOR CAR The owner of a 1953 Hudson Hornet convertible sold the car at an auction. The owner bought it in 1984 when its value was \$11,000. The value of the car increased at a rate of 6.9% per year.

- a. Write a function that models the value of the car over time.
- b. The auction took place in 2004. What was the approximate value of the car at the time of the auction? Round your answer to the nearest dollar.



Solution

- a. Let C be the value of the car (in dollars), and let t be the time (in years) since 1984. The initial value a is \$11,000, and the growth rate r is 0.069.

$$\begin{aligned} C &= a(1 + r)^t && \text{Write exponential growth model.} \\ &= 11,000(1 + 0.069)^t && \text{Substitute 11,000 for } a \text{ and 0.069 for } r. \\ &= 11,000(1.069)^t && \text{Simplify.} \end{aligned}$$

- b. To find the value of the car in 2004, 20 years after 1984, substitute 20 for t .

$$\begin{aligned} C &= 11,000(1.069)^{20} && \text{Substitute 20 for } t. \\ &\approx 41,778 && \text{Use a calculator.} \end{aligned}$$

► In 2004 the value of the car was about \$41,778.

at classzone.com

COMPOUND INTEREST **Compound interest** is interest earned on both an initial investment and on previously earned interest. Compounding of interest can be modeled by exponential growth where a is the initial investment, r is the annual interest rate, and t is the number of years the money is invested.



EXAMPLE 5 Standardized Test Practice

You put \$250 in a savings account that earns 4% annual interest compounded yearly. You do not make any deposits or withdrawals. How much will your investment be worth in 5 years?

- (A) \$300 (B) \$304.16 (C) \$1344.56 (D) \$781,250

Solution

$$\begin{aligned}
 y &= a(1 + r)^t && \text{Write exponential growth model.} \\
 &= 250(1 + 0.04)^5 && \text{Substitute 250 for } a, 0.04 \text{ for } r, \text{ and 5 for } t. \\
 &= 250(1.04)^5 && \text{Simplify.} \\
 &\approx 304.16 && \text{Use a calculator.}
 \end{aligned}$$

You will have \$304.16 in 5 years.

▶ The correct answer is B. (A) (B) (C) (D)

ESTIMATE

You can use the simple interest formula, $I = prt$, to estimate the amount of interest earned: $(250)(0.04)(5) = 50$. Compounding interest will result in slightly more than \$50.



GUIDED PRACTICE for Examples 4 and 5

- WHAT IF?** In Example 4, suppose the owner of the car sold it in 1994. Find the value of the car to the nearest dollar. **\$21,437**
- WHAT IF?** In Example 5, suppose the annual interest rate is 3.5%. How much will your investment be worth in 5 years? **\$296.92**

8.5 EXERCISES

HOMEWORK KEY

- = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 13 and 41
- = **STANDARDIZED TEST PRACTICE**
Exs. 3, 8, 34, 35, 42, 43, 46, and 50
- = **MULTIPLE REPRESENTATIONS**
Ex. 41

SKILL PRACTICE

- VOCABULARY** In the exponential growth model $y = a(1 + r)^t$, the quantity $1 + r$ is called the ? . **growth factor**
- VOCABULARY** For what values of b does the exponential function $y = ab^x$ (where $a > 0$) represent exponential growth? **$b > 1$**
- ★ WRITING** How does the graph of $y = 2 \cdot 5^x$ compare with the graph of $y = 5^x$? *Explain.* **The graph would be a vertical stretch. Sample answer: Since the y -values of $y = 2 \cdot 5^x$ are double those of $y = 5^x$.**

8.5 Write and Graph Exponential Growth Functions **523**

Differentiated Instruction

Advanced Suggest that students research several banks, either locally or online, for interest rates on savings accounts, money market accounts, and CDs. Have them develop exponential growth models for each type of account at each bank on an initial deposit of \$500 after 2 years. Ask students to make a chart comparing the rates and potential earnings for the different types of accounts and the different banks.

See also the *Algebra 1 Toolkit* for more strategies.

Extra Example 5

You put \$125 in a savings account that earns 2% interest compounded yearly. You do not make any deposits or withdrawals. How much will your investment be worth in 5 years? **B**

- (A) \$130 (B) \$138.01
(C) \$311.04 (D) \$4000

Key Question to Ask for Example 5

- Why is an estimate using the simple interest formula useful? **Since it shows that simple interest is \$50 and compound interest would be slightly more than \$50, you can eliminate all answer choices except B.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you write and graph equations for exponential growth functions?

- An exponential function has the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.**
- The exponential growth model is $y = a(1 + r)^t$, where a is the initial amount, r is the growth rate, and t is the time period.**

Use the exponential growth model to write an equation. Substitute the initial amount for a , the rate for r , written as a decimal, and the time period for t . When graphing an exponential function, make a table of negative and positive values, determine the y -intercept, plot the points on a coordinate plane, and connect them with a smooth curve.