8.5 Write and Graph Exponential Growth Functions

Before You wrote and graphed linear models.
Now You will write and graph exponential growth models.
Why? So you can find the value of a collector car, as in Example 4.

Key Vocabulary
- **exponential function**
- **exponential growth**
- **compound interest**

An **exponential function** is a function of the form $y = ab^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$. Exponential functions are *nonlinear* functions. Observe how an exponential function compares with a linear function.

Linear function: $y = 3x + 2$

x	−2	−1	0	1	2
y	−4	−1	2	5	8

(+1 each step; +3 each step)

Exponential function: $y = 2 \cdot 3^x$

x	−2	−1	0	1	2
y	$\frac{2}{9}$	$\frac{2}{3}$	2	6	18

(+1 each step; ×3 each step)

EXAMPLE 1 Write a function rule

Write a rule for the function.

x	−2	−1	0	1	2
y	2	4	8	16	32

Solution

STEP 1 **Tell** whether the function is exponential.

x	−2	−1	0	1	2
y	2	4	8	16	32

(+1 each step; ×2 each step)

Here, the y-values are multiplied by 2 for each increase of 1 in x, so the table represents an exponential function of the form $y = ab^x$ where $b = 2$.

STEP 2 **Find** the value of a by finding the value of y when $x = 0$. When $x = 0$, $y = ab^0 = a \cdot 1 = a$. The value of y when $x = 0$ is 8, so $a = 8$.

STEP 3 **Write** the function rule. A rule for the function is $y = 8 \cdot 2^x$.

GUIDED PRACTICE for Example 1

1. Write a rule for the function.
   $y = 27 \cdot 3^x$

x	−2	−1	0	1	2
y	3	9	27	81	243

1 PLAN AND PREPARE

Warm-Up Exercises

Transparency Available

Write the percent as a decimal.

1. 2% 0.02
2. 5.5% 0.055
3. The table shows the cost of tickets for a matinee. Write a rule for the function. $c = 2t$

Tickets, t	2	4	6	8
Cost, c	4	8	12	16

Notetaking Guide

Transparency Available

Promotes interactive learning and notetaking skills, pp. 179–182.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
• See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3, p. 487

How do you write and graph equations for exponential growth functions? **Tell students they will learn how to answer this question by using models to write functions and tables to graph functions.**

NCTM STANDARDS

Standard 2: Understand patterns; Understand functions

Resource Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan (pp. 47–48)
- Activity Master (p. 49)
- Practice levels A, B, C (pp. 51–56)
- Study Guide (pp. 57–58)
- Catch-up for Absent Students (p. 59)
- Problem Solving Workshop (p. 60)
- Challenge (p. 61)

Workbooks
- Notetaking Guide (pp. 179–182)
- Practice Workbook (pp. 127–129)

Teaching Options
- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

Interactive Technology
- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

Resources for English Learners
- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the *Algebra 1 Toolkit* for more strategies for meeting individual needs.

EXAMPLE 2 Graph an exponential function

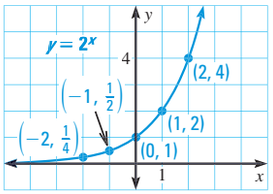
Graph the function $y = 2^x$. Identify its domain and range.

Solution

READ A GRAPH
Notice that the graph has a y-intercept of 1 and that it gets closer to the negative x-axis as the x-values decrease.

STEP 1 **Make** a table by choosing a few values for x and finding the values of y. The domain is all real numbers.

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



STEP 2 **Plot** the points.

STEP 3 **Draw** a smooth curve through the points. From either the table or the graph, you can see that the range is all positive real numbers.

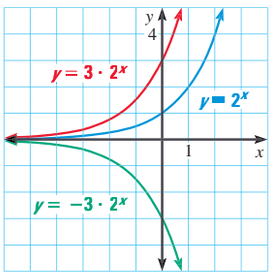
EXAMPLE 3 Compare graphs of exponential functions

Graph the functions $y = 3 \cdot 2^x$ and $y = -3 \cdot 2^x$. Compare each graph with the graph of $y = 2^x$.

Solution

To graph each function, make a table of values, plot the points, and draw a smooth curve through the points.

x	$y = 2^x$	$y = 3 \cdot 2^x$	$y = -3 \cdot 2^x$
-2	$\frac{1}{4}$	$\frac{3}{4}$	$-\frac{3}{4}$
-1	$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$
0	1	3	-3
1	2	6	-6
2	4	12	-12



Because the y-values for $y = 3 \cdot 2^x$ are 3 times the corresponding y-values for $y = 2^x$, the graph of $y = 3 \cdot 2^x$ is a vertical stretch of the graph of $y = 2^x$.

Because the y-values for $y = -3 \cdot 2^x$ are -3 times the corresponding y-values for $y = 2^x$, the graph of $y = -3 \cdot 2^x$ is a vertical stretch with a reflection in the x-axis of the graph of $y = 2^x$.

GUIDED PRACTICE for Examples 2 and 3

2–4. See margin for art.

2. Graph $y = 5^x$ and identify its domain and range.
domain: all real numbers, range: all positive real numbers
3. Graph $y = \frac{1}{3} \cdot 2^x$. Compare the graph with the graph of $y = 2^x$.
The graph is a vertical shrink of the graph of $y = 2^x$.
4. Graph $y = -\frac{1}{3} \cdot 2^x$. Compare the graph with the graph of $y = 2^x$.
The graph is a vertical shrink with a reflection in the x–axis of the graph of $y = 2^x$.

Differentiated Instruction

Inclusion Students with fine-motor problems may find it difficult to graph exponential functions and thus get frustrated. Have these students understand the general shape of an exponential function and have them graph on a coordinate plane without grid lines. Also show how a spreadsheet program or graphing calculator can be used to generate a graph.

See also the *Algebra 1 Toolkit* for more strategies.

Motivating the Lesson

You are studying population growth in your city. You know the current population is 128,256 and the annual rate of growth averages 3%. By knowing how to write and evaluate an exponential growth model, you can estimate the population in 2012.

3 TEACH

Extra Example 1

Write a rule for the function.

x	-2	-1	0	1	2
y	1	4	16	64	256

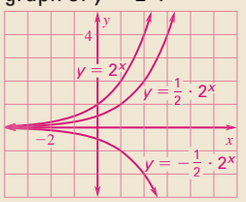
$y = 16 \cdot 4^x$

Extra Example 2

Graph the function $y = 3^x$. Identify its domain and range. **Domain: all real numbers; Range: all positive real numbers**

Extra Example 3

Graph $y = \frac{1}{2} \cdot 2x$ and $y = -\frac{1}{2} \cdot 2x$. Compare each graph with the graph of $y = 2^x$.



The graph of $y = \frac{1}{2} \cdot 2^x$ is a vertical shrink of the graph of $y = 2^x$. The graph of $y = -\frac{1}{2} \cdot 2^x$ is a vertical shrink and reflection in the x-axis of the graph of $y = 2^x$.

2–4. See Additional Answers beginning on p. AA1.