Extra Example 5

The order of magnitude of the luminosity of the Milky Way galaxy is 10^{36} watts. The order of magnitude of the luminosity of a gamma ray burster is 10^{45} watts. How many times as luminous is a gamma ray burster as the Milky Way galaxy? **about 10^9 times as luminous**

Key Question to Ask for Example 5

- How do you know that you need to divide to find the solution to this problem? **The problem asks for a ratio and you divide to find ratios.**

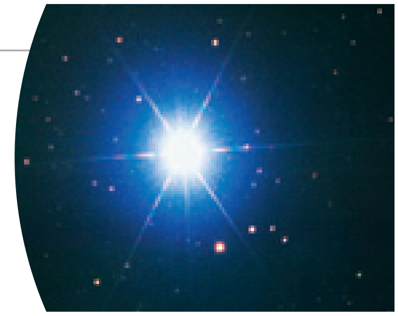
Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you use properties of exponents involving quotients?

- **Subtract exponents to divide powers.**
- **To find the power of a quotient, find the powers of the numerator and the powers of denominator before dividing.**

To simplify an expression involving quotients, subtract the exponent of the denominator from the exponent of the numerator. To raise a quotient to a power, find the power of the numerator and the power of the denominator and divide.

EXAMPLE 5 Solve a real-world problem


Canopus

ASTRONOMY The luminosity (in watts) of a star is the total amount of energy emitted from the star per unit of time. The order of magnitude of the luminosity of the sun is 10^{26} watts. The star Canopus is one of the brightest stars in the sky. The order of magnitude of the luminosity of Canopus is 10^{30} watts. How many times more luminous is Canopus than the sun?

Solution

$$\frac{\text{Luminosity of Canopus (watts)}}{\text{Luminosity of the sun (watts)}} = \frac{10^{30}}{10^{26}} = 10^{30-26} = 10^4$$

▸ Canopus is about 10^4 times as luminous as the sun.

✓ GUIDED PRACTICE for Example 5

10. **WHAT IF?** Sirius is considered the brightest star in the sky. Sirius is less luminous than Canopus, but Sirius appears to be brighter because it is much closer to Earth. The order of magnitude of the luminosity of Sirius is 10^{28} watts. How many times more luminous is Canopus than Sirius? **10^2**

8.2 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 33 and 51
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 37, 46, and 54
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 49

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: In the power 4^3, 4 is the __?__ and 3 is the __?__. **base, exponent**

2. ★ **WRITING** *Explain* when and how to use the quotient of powers property. **When powers have the same base, their quotient is the base raised to the difference of the exponents.**

EXAMPLES 1 and 2 on pp. 495–496 for Exs. 3–20

SIMPLIFYING EXPRESSIONS Simplify the expression. Write your answer using exponents.

3. $\frac{5^6}{5^2}$ **5^4**
4. $\frac{2^{11}}{2^6}$ **2^5**
5. $\frac{3^9}{3^5}$ **3^4**
6. $\frac{(-6)^8}{(-6)^5}$ **$(-6)^3$**
7. $\frac{(-4)^7}{(-4)^4}$ **$(-4)^3$**
8. $\frac{(-12)^9}{(-12)^3}$ **$(-12)^6$**
9. $\frac{10^5 \cdot 10^5}{10^4}$ **10^6**
10. $\frac{6^7 \cdot 6^4}{6^6}$ **6^5**
11. $\left(\frac{1}{3}\right)^5$ **$\frac{1}{3^5}$**
12. $\left(\frac{3}{2}\right)^4$ **$\frac{3^4}{2^4}$**
13. $\left(-\frac{5}{4}\right)^4$ **$\frac{5^4}{4^4}$**
14. $\left(-\frac{2}{5}\right)^5$ **$-\frac{2^5}{5^5}$**
15. $7^9 \cdot \frac{1}{7^2}$ **7^7**
16. $\frac{1}{9^5} \cdot 9^{11}$ **9^6**
17. $\left(\frac{1}{3}\right)^4 \cdot 3^{12}$ **3^8**
18. $4^9 \cdot \left(-\frac{1}{4}\right)^5$ **-4^4**

Differentiated Instruction

Advanced In **Example 5**, some students may see that if Canopus is 10,000 times as bright as the sun, then the sun is $\frac{1}{10{,}000}$ as bright as Canopus. Challenge students to use this observation to write a ratio of the luminosity of the sun to the luminosity of Canopus using exponents. Ask them to explain their reasoning by describing the relationship between the two ratios. This prepares students for the definition of negative exponents in Lesson 8.3.

See also the *Algebra 1 Toolkit* for more strategies.

19. ★ **MULTIPLE CHOICE** Which expression is equivalent to 16^6? **C**

Ⓐ $\frac{16^4}{16^2}$ Ⓑ $\frac{16^{12}}{16^2}$ Ⓒ $\left(\frac{16^6}{16^3}\right)^2$ Ⓓ $\left(\frac{16^9}{16^6}\right)^3$

20. **ERROR ANALYSIS** *Describe* and correct the error in simplifying $\frac{9^5 \cdot 9^3}{9^4}$. **See margin.**

$$\frac{9^5 \cdot 9^3}{9^4} = \frac{9^8}{9^4} = 9^{12} \quad ✗$$

EXAMPLES 1, 2, and 3 on pp. 495–496 for Exs. 21–37

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

21. $\frac{1}{y^8} \cdot y^{15}$ **y^7** 22. $z^8 \cdot \frac{1}{z^7}$ **z** 23. $\left(\frac{a}{y}\right)^9$ **$\frac{a^9}{y^9}$** 24. $\left(\frac{j}{k}\right)^{11}$ **$\frac{j^{11}}{k^{11}}$**

25. $\left(\frac{p}{q}\right)^4$ **$\frac{p^4}{q^4}$** 26. $\left(-\frac{1}{x}\right)^5$ **$-\frac{1}{x^5}$** 27. $\left(-\frac{4}{x}\right)^3$ **$-\frac{64}{x^3}$** 28. $\left(-\frac{a}{b}\right)^4$ **$\frac{a^4}{b^4}$**

29. $\left(\frac{4c}{d^2}\right)^3$ **$\frac{64c^3}{d^6}$** 30. $\left(\frac{a^7}{2b}\right)^5$ **$\frac{a^{35}}{32b^5}$** 31. $\left(\frac{x^2}{3y^3}\right)^2$ **$\frac{x^4}{9y^6}$** 32. $\left(\frac{3x^5}{7y^2}\right)^3$ **$\frac{27x^{15}}{343y^6}$**

33. $\left(\frac{3x^3}{2y}\right)^2 \cdot \frac{1}{x^2}$ **$\frac{9x^4}{4y^2}$** 34. $\left(\frac{2x^3}{y}\right)^3 \cdot \frac{1}{6x^3}$ **$\frac{4x^6}{3y^3}$** 35. $\frac{3}{8m^5} \cdot \left(\frac{m^4}{n^2}\right)^3$ **$\frac{3m^7}{8n^6}$** 36. $\left(-\frac{5}{x}\right)^2 \cdot \left(\frac{2x^4}{y^3}\right)^2$ **$\frac{100x^6}{y^6}$**

B 37. ★ **MULTIPLE CHOICE** Which expression is equivalent to $\left(\frac{7x^3}{2y^4}\right)^2$? **D**

Ⓐ $\frac{7x^5}{2y^6}$ Ⓑ $\frac{7x^6}{2y^8}$ Ⓒ $\frac{49x^5}{4y^6}$ Ⓓ $\frac{49x^6}{4y^8}$

SIMPLIFYING EXPRESSIONS **Find the missing exponent.**

38. $\frac{(-8)^7}{(-8)^?} = (-8)^3$ **4** 39. $\frac{7^? \cdot 7^2}{7^4} = 7^6$ **8** 40. $\frac{1}{p^5} \cdot p^? = p^9$ **14** 41. $\left(\frac{2c^3}{d^2}\right)^? = \frac{16c^{12}}{d^8}$ **4**

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

42. $\left(\frac{2f^2g^3}{3fg}\right)^4$ **$\frac{16f^4g^8}{81}$** 43. $\frac{2s^3t^3}{st^2} \cdot \frac{(3st)^3}{s^2t}$ **$54s^3t^3$** 44. $\left(\frac{2m^5n}{4m^2}\right)^2 \cdot \left(\frac{mn^4}{5n}\right)^2$ **$\frac{m^8n^8}{100}$** 45. $\left(\frac{3x^3y}{x^2}\right)^3 \cdot \left(\frac{y^2x^4}{5y}\right)^2$ **$\frac{27x^{11}y^5}{25}$**

46. ★ **OPEN-ENDED** Write three expressions involving quotients that are equivalent to 14^7. ***Sample answer:*** **$\frac{14^8}{14}, \frac{14^{10}}{14^3}, \frac{14^{14}}{14^7}$**

C 47. **REASONING** Name the definition or property that justifies each step to show that $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ for $m < n$.

Let $m < n$. Given

$\frac{a^m}{a^n} = \frac{a^m}{a^n}\left(\frac{\frac{1}{a^m}}{\frac{1}{a^m}}\right)$? **Identity property of multiplication**

$= \frac{1}{\frac{a^n}{a^m}}$? **Multiply fractions.**

$= \frac{1}{a^{n-m}}$? **Quotient of powers property**

48. **CHALLENGE** Find the values of x and y if you know that $\frac{b^x}{b^y} = b^9$ and $\frac{b^x \cdot b^2}{b^{3y}} = b^{13}$. *Explain* how you found your answer.

48. $x = 8$, $y = -1$. *Sample answer:* Using the quotient of a power property, write two equations for x and y: $x - y = 9$, and $x + 2 - 3y = 13$. Solve the equations.

20. *Sample answer:* When using the quotient of powers property, the base is raised to the difference of the exponents, not the sum; $\frac{9^8}{9^4} = 9^{(8-4)} = 9^4$.

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: pp. 498–501
Exs. 1–28
Day 2: pp. 498–501
Exs. 29–39, 49–52, 55–66

Average:
Day 1: pp. 498–501
Exs. 1, 2, 7–28, 38–40
Day 2: pp. 498–501
Exs. 31–37, 41–46, 49–53, 55–66

Advanced:
Day 1: pp. 498–501
Exs. 1, 2, 9–19, 21–28, 38–40, 47, 48*
Day 2: pp. 498–501
Exs. 32–37, 41–46, 49–66*

Block:
pp. 498–501
Exs. 1, 2, 7–28, 38–40 (with 8.1)
pp. 498–501
Exs. 31–37, 41–46, 49–53, 55–66 (with 8.3)

Differentiated Instruction

See *Algebra 1 Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 6, 23, 30, 49, 50
Average: 12, 24, 32, 49, 51
Advanced: 16, 27, 34, 50, 51

Extra Practice

- Student Edition, p. 945
- Chapter 8 Resource Book: Practice levels A, B, C, pp. 17–19

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 486C.