

# 8.2 Apply Exponent Properties Involving Quotients



- Before** You used properties of exponents involving products.
- Now** You will use properties of exponents involving quotients.
- Why?** So you can compare magnitudes of earthquakes, as in Ex. 53.

### Key Vocabulary

- power, p. 3
- exponent, p. 3
- base, p. 3

Notice what happens when you divide powers with the same base.

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a \cdot a = a^2 = a^{5-3}$$

The example above suggests the following property of exponents, known as the quotient of powers property.

### KEY CONCEPT

*For Your Notebook*

#### Quotient of Powers Property

Let  $a$  be a nonzero real number, and let  $m$  and  $n$  be positive integers such that  $m > n$ .

**Words** To divide powers having the same base, subtract exponents.

**Algebra**  $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$       **Example**  $\frac{4^7}{4^2} = 4^{7-2} = 4^5$

### EXAMPLE 1 Use the quotient of powers property

a.  $\frac{8^{10}}{8^4} = 8^{10-4}$   
 $= 8^6$

b.  $\frac{(-3)^9}{(-3)^3} = (-3)^{9-3}$   
 $= (-3)^6$

c.  $\frac{5^4 \cdot 5^8}{5^7} = \frac{5^{12}}{5^7}$   
 $= 5^{12-7}$   
 $= 5^5$

d.  $\frac{1}{x^4} \cdot x^6 = \frac{x^6}{x^4}$   
 $= x^{6-4}$   
 $= x^2$

### SIMPLIFY EXPRESSIONS

When simplifying powers with numerical bases only, write your answers using exponents, as in parts (a), (b), and (c).

### GUIDED PRACTICE for Example 1

Simplify the expression.

1.  $\frac{6^{11}}{6^5} \cdot 6^6$

2.  $\frac{(-4)^9}{(-4)^2} (-4)^7$

3.  $\frac{9^4 \cdot 9^3}{9^2} 9^5$

4.  $\frac{1}{y^5} \cdot y^8 \cdot y^3$

## 1 PLAN AND PREPARE

### Warm-Up Exercises

**Transparency Available**  
Evaluate the expression.

1.  $q^3$  when  $q = \frac{1}{4} \cdot \frac{1}{64}$

2.  $c^2$  when  $c = \frac{3}{5} \cdot \frac{9}{25}$

3. A magazine had a circulation of 9364 in 2001. The circulation was about 125 times greater in 2006. Use order of magnitude to estimate the circulation in 2006. **about  $10^6$  or 1,000,000**

### Notetaking Guide

**Transparency Available**  
Promotes interactive learning and notetaking skills, pp. 170–172.

### Pacing

**Basic:** 2 days

**Average:** 2 days

**Advanced:** 2 days

**Block:** 0.5 block with 8.1

• See *Teaching Guide/Lesson Plan*.

## 2 FOCUS AND MOTIVATE

### Essential Question

**Big Idea 1, p. 487**

How do you use properties of exponents involving quotients?

**Tell students they will learn how to answer this question by simplifying expressions that involve division of powers.**

### NCTM STANDARDS

**Standard 1:** Understand how operations are related

**Standard 2:** Understand patterns

## Resource Planning Guide

### Chapter Resource Book

- Teaching Guide/Lesson Plan (pp. 14–15)
- Activity Master (p. 16)
- Practice levels A, B, C (pp. 17–19)
- Study Guide (pp. 20–21)
- Catch-up for Absent Students (p. 22)
- Application (p. 23)
- Challenge (p. 24)

### Workbooks

- Notetaking Guide (pp. 170–172)
- Practice Workbook (pp. 121–122)

### Teaching Options

- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

### Interactive Technology

- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

### Resources for English Learners

- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the *Algebra 1 Toolkit* for more strategies for meeting individual needs.

## Motivating the Lesson

You are preparing a report that compares the radius of Earth at an order of magnitude of  $10^7$  meters to the radius of the Milky Way galaxy at an order of magnitude of  $10^{21}$  meters. By knowing how to apply properties of exponents that involve quotients, you will be able to approximate how many times as great the radius of the Milky Way galaxy is as the radius of Earth.

## 3 TEACH

### Extra Example 1

Use the quotient of powers property.

a.  $\frac{9^{12}}{9^5}$   **$9^7$**

b.  $\frac{(-2)^4}{(-2)^3}$   **$-2$**

c.  $\frac{6^3 \cdot 6^4}{6^2}$   **$6^5$**

d.  $\frac{1}{r^5} \cdot r^8$   **$r^3$**

### Key Question to Ask for Example 1

- How is dividing powers different from multiplying powers? **You subtract exponents when you divide powers and add exponents when you multiply powers.**

### Extra Example 2

Use the power of a quotient property.

a.  $\left(\frac{c}{d}\right)^6$   **$\frac{c^6}{d^6}$**

b.  $\left(\frac{-2}{y}\right)^4$   **$\frac{16}{y^4}$**

**POWER OF A QUOTIENT** Notice what happens when you raise a quotient to a power.

$$\left(\frac{a}{b}\right)^4 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} = \frac{a \cdot a \cdot a \cdot a}{b \cdot b \cdot b \cdot b} = \frac{a^4}{b^4}$$

The example above suggests the following property of exponents, known as the power of a quotient property.

### KEY CONCEPT

*For Your Notebook*

#### Power of a Quotient Property

Let  $a$  and  $b$  be real numbers with  $b \neq 0$ , and let  $m$  be a positive integer.

**Words** To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

**Algebra**  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ ,  $b \neq 0$

**Example**  $\left(\frac{3}{2}\right)^7 = \frac{3^7}{2^7}$

### SIMPLIFY EXPRESSIONS

When simplifying powers with numerical and variable bases, evaluate the numerical power, as in part (b).

### EXAMPLE 2 Use the power of a quotient property

a.  $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

b.  $\left(\frac{-7}{x}\right)^2 = \frac{(-7)^2}{x^2} = \frac{49}{x^2}$

### EXAMPLE 3 Use properties of exponents

a.  $\left(\frac{4x^2}{5y}\right)^3 = \frac{(4x^2)^3}{(5y)^3}$  **Power of a quotient property**

$= \frac{4^3 \cdot (x^2)^3}{5^3 y^3}$  **Power of a product property**

$= \frac{64x^6}{125y^3}$  **Power of a power property**

b.  $\left(\frac{a^2}{b}\right)^5 \cdot \frac{1}{2a^2} = \frac{(a^2)^5}{b^5} \cdot \frac{1}{2a^2}$  **Power of a quotient property**

$= \frac{a^{10}}{b^5} \cdot \frac{1}{2a^2}$  **Power of a power property**

$= \frac{a^{10}}{2a^2 b^5}$  **Multiply fractions.**

$= \frac{a^8}{2b^5}$  **Quotient of powers property**

### Differentiated Instruction

**Inclusion** For smaller exponents, explicitly writing out the factors of each power works for division as well as multiplication. As practice, students can attempt a similar problem with very large exponents. This will allow them to see why the quotient of powers property works, without having to memorize its verbal form. See also the *Algebra 1 Toolkit* for more strategies.

**GUIDED PRACTICE** for Examples 2 and 3

Simplify the expression.

5.  $\left(\frac{a}{b}\right)^2 \frac{a^2}{b^2}$

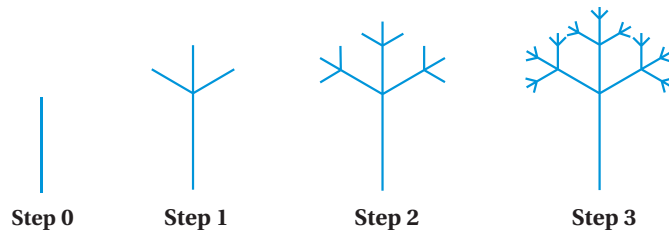
6.  $\left(-\frac{5}{y}\right)^3 - \frac{125}{y^3}$

7.  $\left(\frac{x^2}{4y}\right)^2 \frac{x^4}{16y^2}$

8.  $\left(\frac{2s}{3t}\right)^3 \cdot \left(\frac{t^5}{16}\right) \frac{s^3 t^2}{54}$

**EXAMPLE 4** Solve a multi-step problem

**FRACTAL TREE** To construct what is known as a *fractal tree*, begin with a single segment (the trunk) that is 1 unit long, as in Step 0. Add three shorter segments that are  $\frac{1}{2}$  unit long to form the first set of branches, as in Step 1. Then continue adding sets of successively shorter branches so that each new set of branches is half the length of the previous set, as in Steps 2 and 3.



- Make a table showing the number of new branches at each step for Steps 1–4. Write the number of new branches as a power of 3.
- How many times greater is the number of new branches added at Step 5 than the number of new branches added at Step 2?

**Solution**

Step	Number of new branches
1	$3 = 3^1$
2	$9 = 3^2$
3	$27 = 3^3$
4	$81 = 3^4$

- The number of new branches added at Step 5 is  $3^5$ . The number of new branches added at Step 2 is  $3^2$ . So, the number of new branches added at Step 5 is  $\frac{3^5}{3^2} = 3^3 = 27$  times the number of new branches added at Step 2.

**GUIDED PRACTICE** for Example 4

- FRACTAL TREE** In Example 4, add a column to the table for the length of the new branches at each step. Write the lengths of the new branches as powers of  $\frac{1}{2}$ . What is the length of a new branch added at Step 9? **See margin.**

Step	Number of new branches	Length of new branches
1	$3 = 3^1$	$\frac{1}{2} = \left(\frac{1}{2}\right)^1$
2	$9 = 3^2$	$\frac{1}{4} = \left(\frac{1}{2}\right)^2$
3	$27 = 3^3$	$\frac{1}{8} = \left(\frac{1}{2}\right)^3$
4	$81 = 3^4$	$\frac{1}{16} = \left(\frac{1}{2}\right)^4$

$\left(\frac{1}{2}\right)^9 = \frac{1}{512}$  unit

**Extra Example 3**

Use properties of exponents.

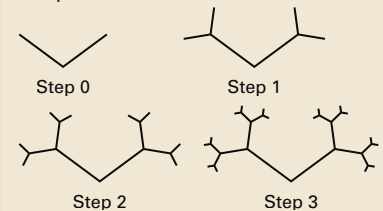
a.  $\left(\frac{3a^4}{5b}\right)^3 \frac{27a^{12}}{125b^9}$

b.  $\left(\frac{x^3}{y}\right)^7 \cdot \frac{1}{3x^8} \frac{x^{13}}{3y^7}$

**Extra Example 4**

Construct a fractal tree that begins with a V-shaped segment, with each side of the V-shape 1 unit long. Then add a V-shaped segment on each end with each side  $\frac{1}{2}$  unit long.

Continue adding sets of successively shorter branches so that each new set of branches is half the length of the previous set.



- Make a table showing the number of new branches at each step for Steps 1–3. Write the number of new branches as a power of 2.

Step	New branches
1	$4 = 2^2$
2	$8 = 2^3$
3	$16 = 2^4$

- How many times as great is the number of new branches at Step 6 as the number of new branches at Step 2? **16**

**Key Question to Ask for Example 4**

- Why do you write the number of new branches as a power of 3?  
**Since you add 3 segments to each set of successive branches, the steps are 3, then  $3 \times 3$ , then  $3 \times 3 \times 3$ , and so on.**