8.2 Apply Exponent Properties Involving Quotients

Before You used properties of exponents involving products.

Now You will use properties of exponents involving quotients.

Why? So you can compare magnitudes of earthquakes, as in Ex. 53.

Key Vocabulary
- **power**, *p. 3*
- **exponent**, *p. 3*
- **base**, *p. 3*

Notice what happens when you divide powers with the same base.

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a \cdot a = a^2 = a^{5-3}$$

The example above suggests the following property of exponents, known as the quotient of powers property.

KEY CONCEPT
For Your Notebook

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be positive integers such that $m > n$.

Words To divide powers having the same base, subtract exponents.

Algebra $\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$ **Example** $\dfrac{4^7}{4^2} = 4^{7-2} = 4^5$

EXAMPLE 1 Use the quotient of powers property

SIMPLIFY EXPRESSIONS
When simplifying powers with numerical bases only, write your answers using exponents, as in parts (a), (b), and (c).

a. $\dfrac{8^{10}}{8^4} = 8^{10-4}$

$= 8^6$

b. $\dfrac{(-3)^9}{(-3)^3} = (-3)^{9-3}$

$= (-3)^6$

c. $\dfrac{5^4 \cdot 5^8}{5^7} = \dfrac{5^{12}}{5^7}$

$= 5^{12-7}$

$= 5^5$

d. $\dfrac{1}{x^4} \cdot x^6 = \dfrac{x^6}{x^4}$

$= x^{6-4}$

$= x^2$

✔ **GUIDED PRACTICE** for Example 1

Simplify the expression.

1. $\dfrac{6^{11}}{6^5}$ **6^6**

2. $\dfrac{(-4)^9}{(-4)^2}$ **$(-4)^7$**

3. $\dfrac{9^4 \cdot 9^3}{9^2}$ **9^5**

4. $\dfrac{1}{y^5} \cdot y^8$ **y^3**

8.2 Apply Exponent Properties Involving Quotients **495**

POWER OF A QUOTIENT Notice what happens when you raise a quotient to a power.

$$\left(\frac{a}{b}\right)^4 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} = \frac{a \cdot a \cdot a \cdot a}{b \cdot b \cdot b \cdot b} = \frac{a^4}{b^4}$$

The example above suggests the following property of exponents, known as the power of a quotient property.

KEY CONCEPT   *For Your Notebook*

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be a positive integer.

Words To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

Algebra $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m},\ b \neq 0$

Example $\left(\dfrac{3}{2}\right)^7 = \dfrac{3^7}{2^7}$

EXAMPLE 2 **Use the power of a quotient property**

SIMPLIFY EXPRESSIONS
When simplifying powers with numerical *and* variable bases, evaluate the numerical power, as in part (b).

a. $\left(\dfrac{x}{y}\right)^3 = \dfrac{x^3}{y^3}$

b. $\left(-\dfrac{7}{x}\right)^2 = \left(\dfrac{-7}{x}\right)^2 = \dfrac{(-7)^2}{x^2} = \dfrac{49}{x^2}$

EXAMPLE 3 **Use properties of exponents**

a. $\left(\dfrac{4x^2}{5y}\right)^3 = \dfrac{(4x^2)^3}{(5y)^3}$ Power of a quotient property

$= \dfrac{4^3 \cdot (x^2)^3}{5^3 y^3}$ Power of a product property

$= \dfrac{64x^6}{125y^3}$ Power of a power property

b. $\left(\dfrac{a^2}{b}\right)^5 \cdot \dfrac{1}{2a^2} = \dfrac{(a^2)^5}{b^5} \cdot \dfrac{1}{2a^2}$ Power of a quotient property

$= \dfrac{a^{10}}{b^5} \cdot \dfrac{1}{2a^2}$ Power of a power property

$= \dfrac{a^{10}}{2a^2 b^5}$ Multiply fractions.

$= \dfrac{a^8}{2b^5}$ Quotient of powers property

Differentiated Instruction

Inclusion For smaller exponents, explicitly writing out the factors of each power works for division as well as multiplication. As practice, students can attempt a similar problem with very large exponents. This will allow them to see why the quotient of powers property works, without having to memorize its verbal form.

See also the *Algebra 1 Toolkit* for more strategies.

Simplify the expression.

5. $\left(\dfrac{a}{b}\right)^2$ $\dfrac{a^2}{b^2}$

6. $\left(-\dfrac{5}{y}\right)^3$ $-\dfrac{125}{y^3}$

7. $\left(\dfrac{x^2}{4y}\right)^2$ $\dfrac{x^4}{16y^2}$

8. $\left(\dfrac{2s}{3t}\right)^3 \cdot \left(\dfrac{t^5}{16}\right)$ $\dfrac{s^3 t^2}{54}$

EXAMPLE 4   **Solve a multi-step problem**

FRACTAL TREE To construct what is known as a *fractal tree*, begin with a single segment (the trunk) that is 1 unit long, as in Step 0. Add three shorter segments that are $\frac{1}{2}$ unit long to form the first set of branches, as in Step 1. Then continue adding sets of successively shorter branches so that each new set of branches is half the length of the previous set, as in Steps 2 and 3.

Step 0   Step 1   Step 2   Step 3
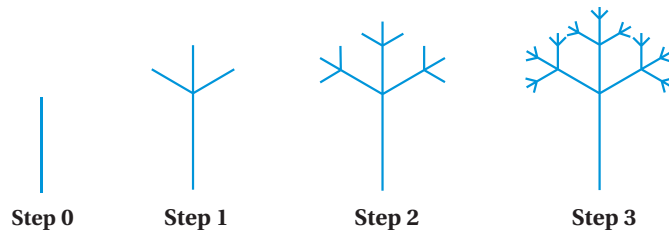
a. Make a table showing the number of new branches at each step for Steps 1–4. Write the number of new branches as a power of 3.

b. How many times greater is the number of new branches added at Step 5 than the number of new branches added at Step 2?

Solution

a.

Step	Number of new branches
1	$3 = 3^1$
2	$9 = 3^2$
3	$27 = 3^3$
4	$81 = 3^4$

b. The number of new branches added at Step 5 is 3^5. The number of new branches added at Step 2 is 3^2. So, the number of new branches added at Step 5 is $\dfrac{3^5}{3^2} = 3^3 = 27$ times the number of new branches added at Step 2.

✓ **GUIDED PRACTICE** for Example 4

9. FRACTAL TREE In Example 4, add a column to the table for the length of the new branches at each step. Write the lengths of the new branches as powers of $\frac{1}{2}$. What is the length of a new branch added at Step 9? **See margin.**

9.

Step	Number of new branches	Length of new branches
1	$3 = 3^1$	$\dfrac{1}{2} = \left(\dfrac{1}{2}\right)^1$
2	$9 = 3^2$	$\dfrac{1}{4} = \left(\dfrac{1}{2}\right)^2$
3	$27 = 3^3$	$\dfrac{1}{8} = \left(\dfrac{1}{2}\right)^3$
4	$81 = 3^4$	$\dfrac{1}{16} = \left(\dfrac{1}{2}\right)^4$

$\left(\dfrac{1}{2}\right)^9 = \dfrac{1}{512}$ unit


Extra Example 3

Use properties of exponents.

a. $\left(\dfrac{3a^4}{5b}\right)^3$ $\dfrac{27a^{12}}{125b^6}$

b. $\left(\dfrac{x^3}{y}\right)^7 \cdot \dfrac{1}{3x^8}$ $\dfrac{x^{13}}{3y^7}$

Extra Example 4

Construct a fractal tree that begins with a V-shaped segment, with each side of the V-shape 1 unit long. Then add a V-shaped segment on each end with each side $\frac{1}{2}$ unit long. Continue adding sets of successively shorter branches so that each new set of branches is half the length of the previous set.

Step 0   Step 1

Step 2   Step 3
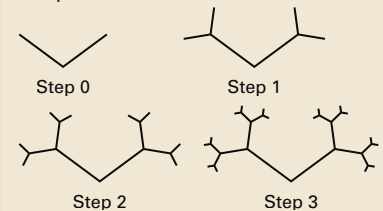
a. Make a table showing the number of new branches at each step for Steps 1–3. Write the number of new branches as a power of 2.

Step	New branches
1	$4 = 2^2$
2	$8 = 2^3$
3	$16 = 2^4$

b. How many times as great is the number of new branches at Step 6 as the number of new branches at Step 2? **16**

Key Question to Ask for Example 4

• Why do you write the number of new branches as a power of 3? **Since you add 3 segments to each set of successive branches, the steps are 3, then 3 × 3, then 3 × 3 × 3, and so on.**